June 19th, 2021 **Name** ____________________

SWMS- Worksheet 5 in Probability and Statistics

1. Using `R`, estimate the following integrals using an enclosing box.

   (a) $\int \mathbb{I}(5x^2 + y^2 < 1)dxdy$

   (b) $\int \mathbb{I}(4x^2 + 3y^2 + 2xy < 1)dxdy$

   (c) $\int_0^1 e^{-x^2/20}dx$

   (d) $\int_0^{10} x^{15}e^{-5x}dx$

After-class and a little hard

Volume of higher-dimensional spheres

We know the area of a circle, and we know the volume of the sphere, but of course, we don't know the volume of a general k-dimensional sphere. Consider the k-sphere

$$x_1^2 + x_2^2 + \cdots + x_k^2 < 1$$

Then the volume of the k-sphere is

$$\int \mathbb{I}(x_1^2 + x_2^2 + \cdots + x_k^2 < 1) dx_1 \ldots dx_k \,.$$

We can estimate the volume of the box using the same idea as before. Enclose the k-sphere in a k-box, obtain $\hat{p}$: the proportion of points in the region and estimate the volume of the sphere.

$$\text{Volume of } k\text{-sphere} = \hat{p} \cdot \text{Volume of the } k\text{-box}\,.$$

Implement this in `R` for $k = 2, 3, 10, 15$.