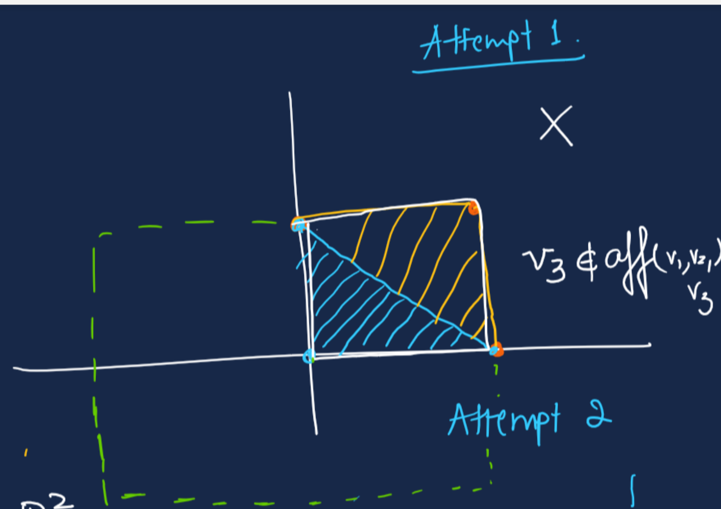
Worksheet 1 continued

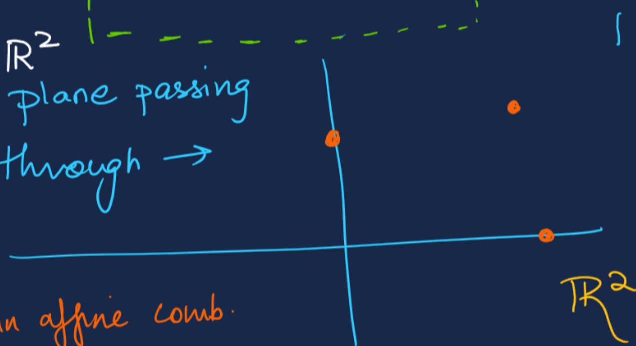
$$\mathrm{aff}\Big(\underset{v_1}{(1,0)},\ \underset{v_2}{(0,1)},\ \underset{v_3}{(1,1)}\Big)$$

$$(1,1) = 0\cdot v_1 + 0\cdot v_2 + 1\cdot v_3$$



✓ Priyanka's solution : Claim $\mathrm{aff}(v_1,v_2,v_3) = \mathbb{R}^2$

plane passing through →



① $\mathrm{aff}(v_1,v_2,v_3) \subseteq \mathbb{R}^2$

② $\mathbb{R}^2 \subseteq \mathrm{aff}(v_1,v_2,v_3)$.

We must show that any $(a,b) \in \mathbb{R}^2$ is an affine comb. of v_1, v_2 & v_3.

Note
$$t_1v_1 + t_2v_2 + t_3v_3$$
$$= (t_1+t_3,\ t_2+t_3)$$
$$= (1-t_2,\ 1-t_1)$$

$$\frac{t_1+t_2+t_3=1}{t_1,t_2,t_3 \in \mathbb{R}}$$

→ $t_1, t_2 \in \mathbb{R}$.

Take any $(a,b) \in \mathbb{R}^2$. Then, $(a,b) = \Big(1-\overbrace{(1-a)}^{t_2},\ 1-\overbrace{(1-b)}^{t_1}\Big)$

$t_3 := 1-t_1-t_2$.

solve : $(a,b) = (1-t_2,\ 1-t_1)$ for t_1, t_2

Kanupriya's claim : $\mathrm{cvx}(v_1,v_2,v_3)$ = square formed by $(1,0)$ $(0,1)$ $(1,1)$ & $(0,0)$.

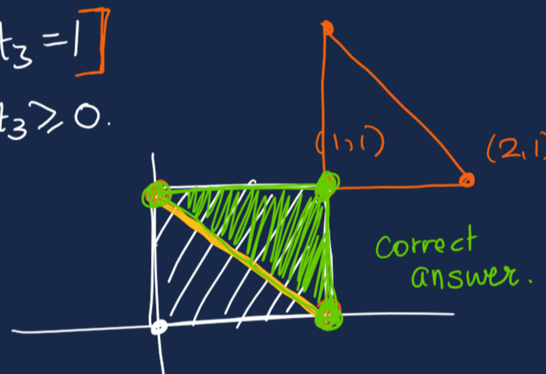
$$t_1v_1 + t_2v_2 + t_3v_3$$
$$\|$$
$$(t_1+t_3,\ t_2+t_3)$$
$$\| \ (*)$$
$$(1-t_2,\ 1-t_1)$$

(*) $t_1+t_2+t_3=1$

(**) $t_1, t_2, t_3 \geq 0$.

X $\boxed{\cancel{0 \leq t_1, t_2 \leq 1}}$

$\boxed{t_1 \geq 0,\ t_2 \geq 0}$ ✓



Nikita's claim.

$0 \leq t_3 = 1-t_1-t_2 \Rightarrow 0 \leq 1-t_1-t_2$

$\Rightarrow \boxed{t_1+t_2 \leq 1}$ ✓

New understanding: $(1-t_2,\ 1-t_1)$ such that $t_1, t_2 \geq 0$, $t_1 + t_2 \leq 1$

new substitution: $s_2 = 1 - t_2$, $s_1 = 1 - t_1$

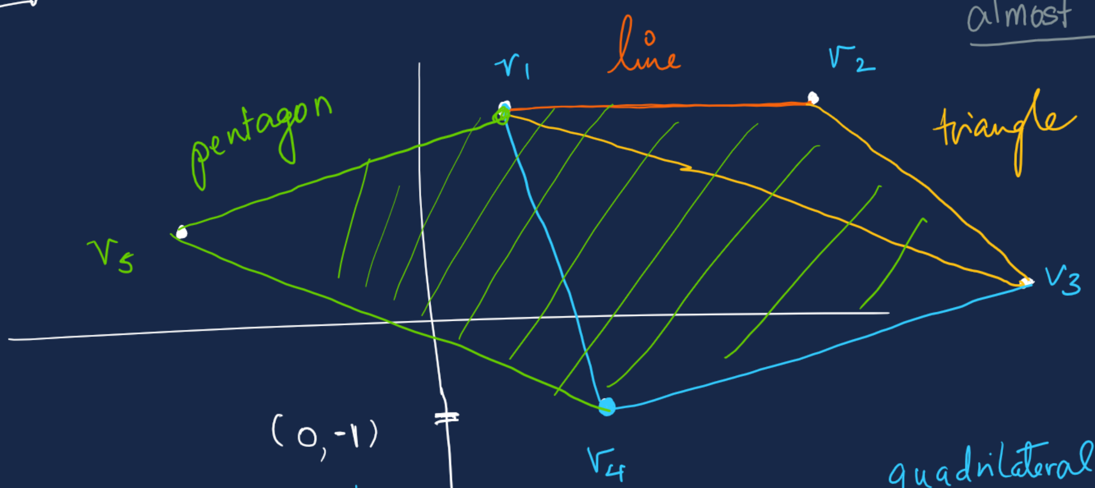
$\Downarrow$ Ex

?

Problem 1f) $\mathrm{cvx}(v_1, \dots, v_k)$ ~ general shap

Guess: k-sided polygon

almost correct



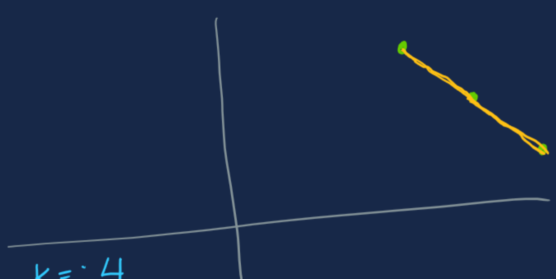
$(0,-1)$

$\downarrow$

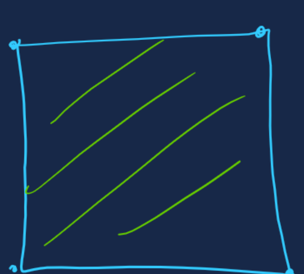
Some components of convex combinations can be negative!!

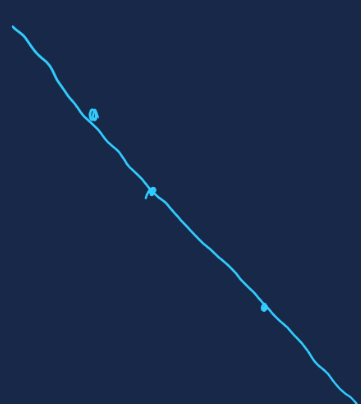
Q. Is the cvx hull of 3 vectors always a triangle?

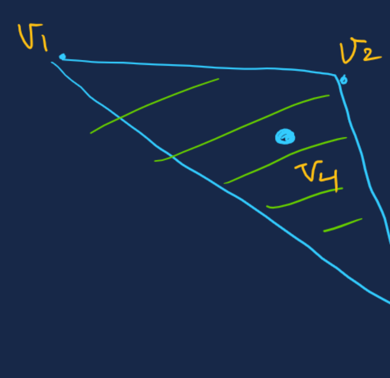
No, not if they are colinear!



K = 4





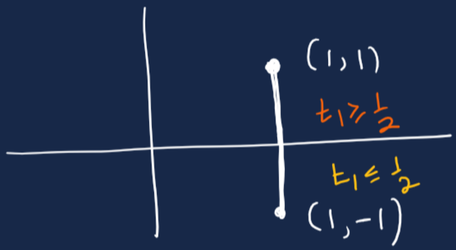


$\mathrm{cvx}(\overbrace{v_1, v_2, v_3}^{\text{fixed}}, v_4)$

"junk"

$\mathrm{cvx}(v_1, v_2, v_3)$

Talks » New Section 1



$\mathrm{cvx}\,\big((1,1),(1,-1)\big) \ni (a,b)$

$(a,b) = t_1(1,1) + t_2(1,-1)\,, \quad t_1, t_2 \geq 0$

$\boxed{t_1 + t_2 = 1}$

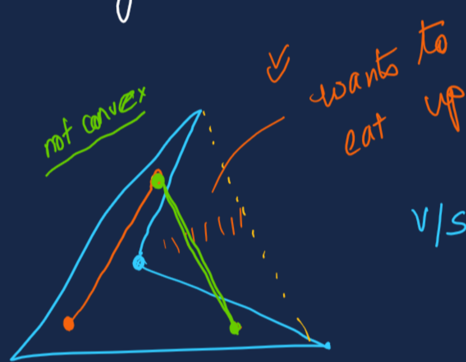
$= (t_1 + t_2\,,\ \underline{t_1 - t_2})$

$= \big(1,\ t_1 - (1 - t_1)\big)$

$= (1,\ 2t_1 - 1) \qquad 0 \leq t_1 \leq 1$

Definition. A $\boxed{\text{convex}}$ polytope in $\mathbb{R}^n$ is the convex hull of finitely many vectors.

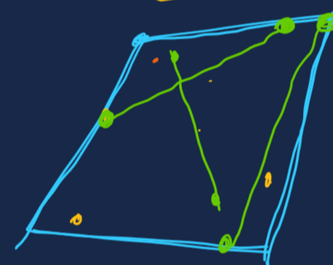
$\boxed{\text{non convex}}$ polygon :



this is never $\mathrm{cvx}(v_1, v_2, v_3, v_4)$.

v/s

$\boxed{\text{convex}}$ polygon



"convexity"

interior angles cannot be $> 180°$

Kanupriya:

Does this have anything to do with linear independence?