1. A sequence $\{a_n\}$ is a bounded sequence if there is a $M > 0$ such that a_n is in the interval $(-M, M)$ for all $n \in \mathbb{N}$.
   (a) Provide an example of a bounded sequence: which converges and which does not converge to a real number.
   (b) Write a logical statement[1] that is equivalent to saying that the sequence a_n is bounded.
   (c) Write a logical statement that is equivalent to saying that the sequence a_n is not bounded.

2. Find an example of a sequence that satisfies the below statements and then write the below statements using logical notation:
   (a) For every $\epsilon > 0$ there *are infinitely many* n such that distance of a_n to 0 is less than ϵ.
   (b) For every $\epsilon > 0$ *for all but finitely many* n the distance of a_n to 0 is less than ϵ.

3. Let $a, b : \mathbb{N} \to \mathbb{R}_+$ be two sequences
   - $a_n = O(b_n)$ if there exists $N_0 \in \mathbb{N}$ and $c > 0$ such that $a_n \leq cb_n$ for all $n \geq N_0$
   - $a_n = o(b_n)$ if for every $\epsilon > 0$ there exists N_0 such that $a_n \leq \epsilon b_n$ for all $n \geq N_0$

   For each of the following indicate whether $a_n = O(b_n)$, or $a_n = o(b_n)$
   (a) $a_n = n^3 + 5n^2 + 15$ and $b_n = n^3 + 7n + 8$
   (b) $a_n = nb^n$, for $b \in (0, 1)$ and $b_n = \frac{1}{n^4}$

[1] **Logical Notation:** • $\forall$ to mean for all; • $\exists$ to mean there exists; • $\implies$ to mean implies; and • $\iff$ to mean equivalent.