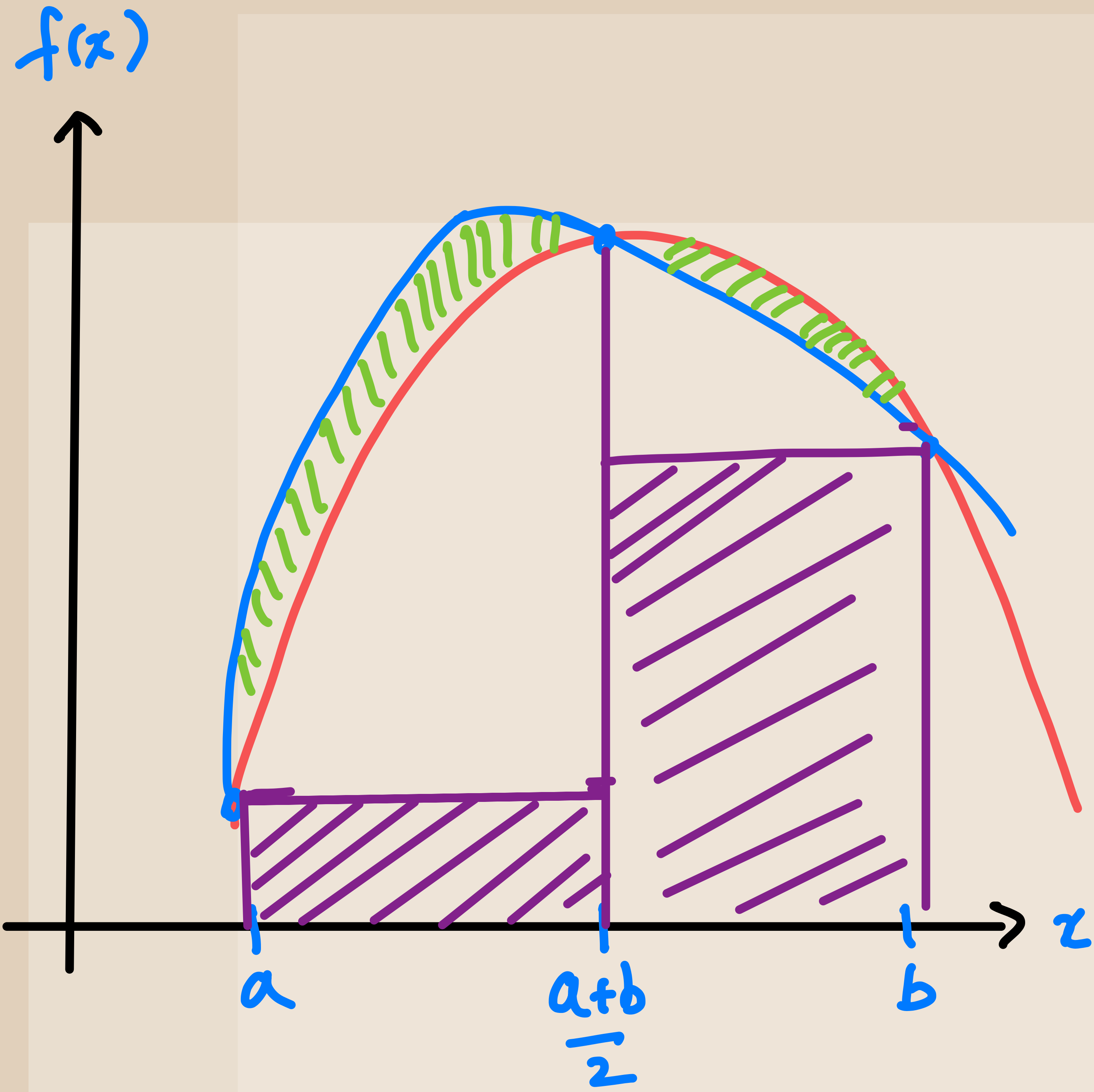
SWMS - 2021

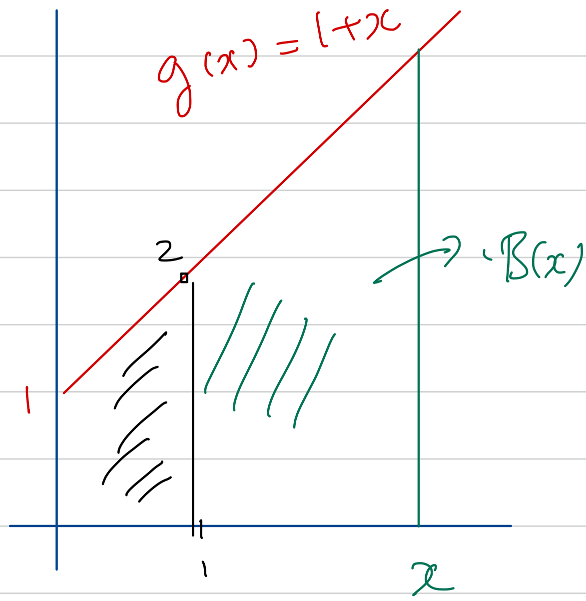
June 14^{th}, 2021



Lecture 1

Integral Approximation

GROUP 1



1 (a) $B(1) = \frac{3}{2}$

$B(3) - B(2) = \frac{7}{2}$

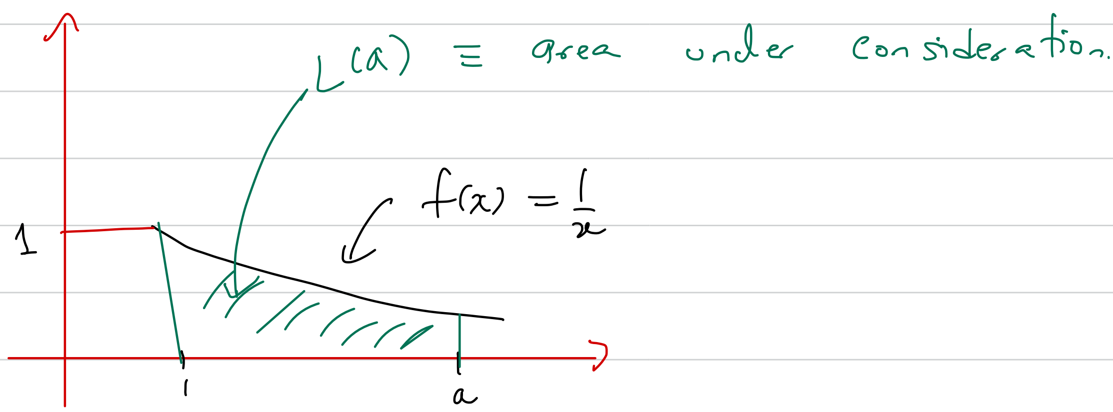
$B(4) = 12$

(b) $$B(x) = \frac{(1 + (1+x))(x)}{2}$$

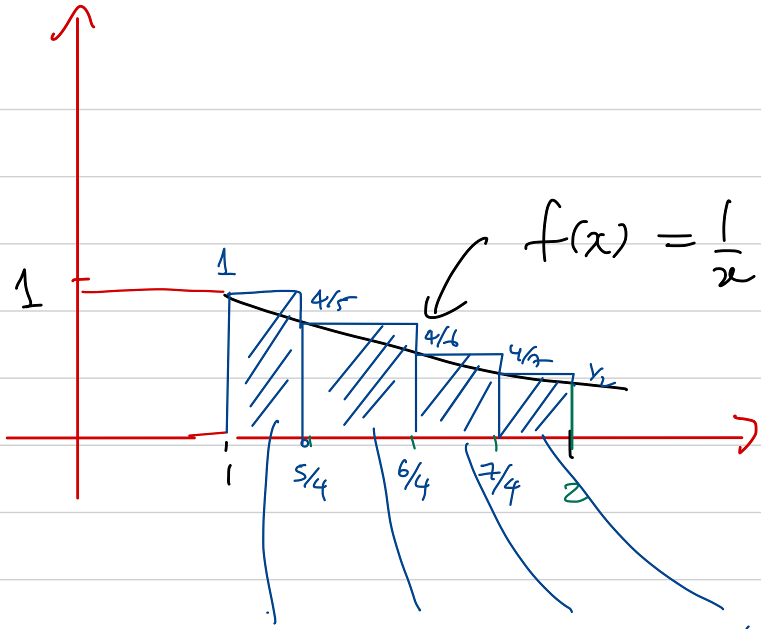
$$= \frac{2x + x^2}{2} = x + \frac{x^2}{2}$$

(Derivative of area) $B'(x) = 1 + x \equiv g(x)$ (function)

2



(a)

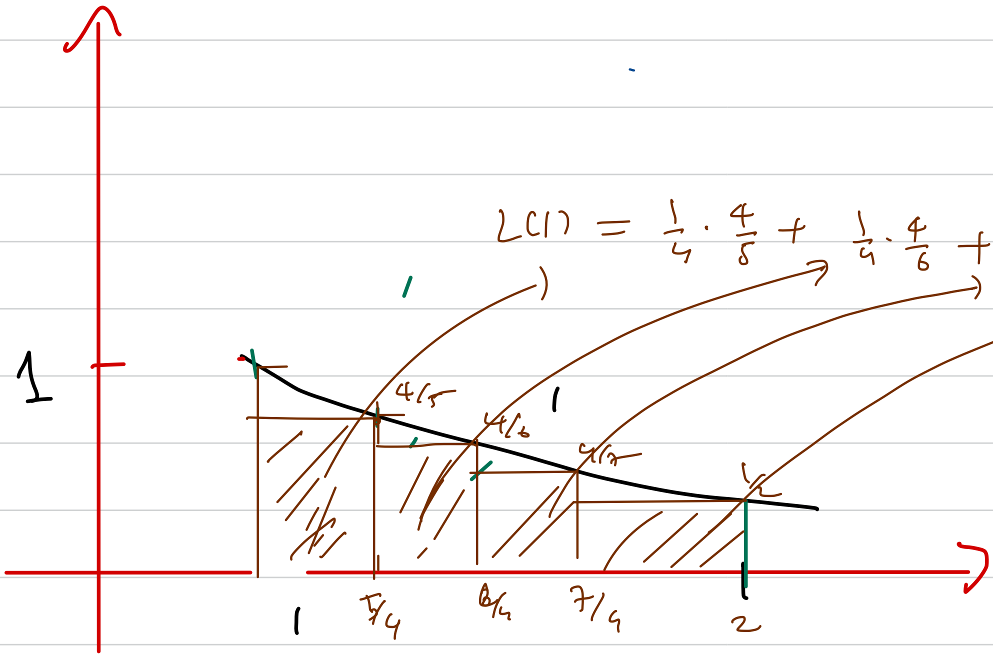


(b) $U(1) = 1 \cdot \frac{1}{4} + \frac{4}{5} \cdot \frac{1}{4} + 4/6 \cdot \frac{1}{4} + \frac{4}{7} \cdot \frac{1}{4} = 0.7595$

(c)

$L(1) = \frac{1}{4} \cdot \frac{4}{5} + \frac{1}{4} \cdot \frac{4}{6} + \frac{1}{4} \cdot \frac{4}{7} + \frac{1}{4} \cdot \frac{1}{2}$

$= 0.6345$



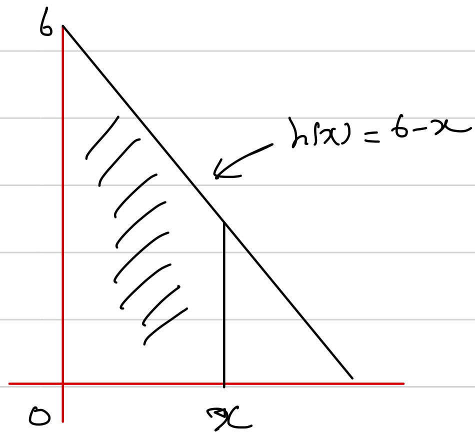
(d) $\frac{L(1) + U(1)}{2} \cong 0.6970$ } (approximation)

$\ln(2) \cong 0.693$

Group 2

1 (a) $\rightarrow C(2) = \dfrac{h(2)+h(0)}{2}(2-0)$

$= \dfrac{(4+6)}{2}\cdot 2 = 10$



$\rightarrow C(3) - C(1) = 8$

$\rightarrow C(4) = 16$

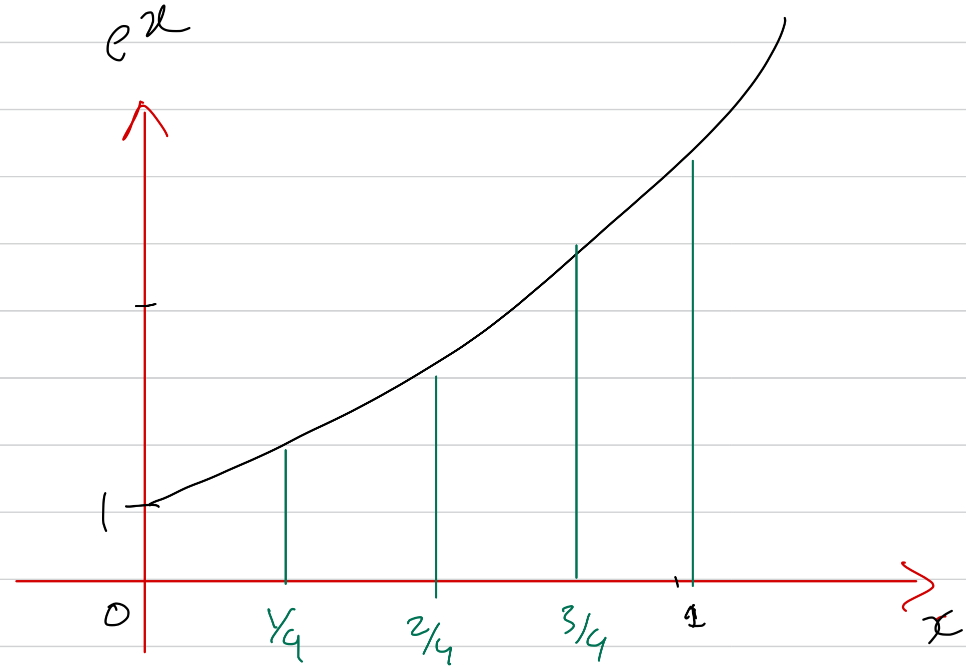
(b) $C(x) = \dfrac{h(x)+h(0)}{2}\cdot(x-0)$

$= \dfrac{(12-x)(x)}{2} = 6x - \dfrac{x^2}{2}$

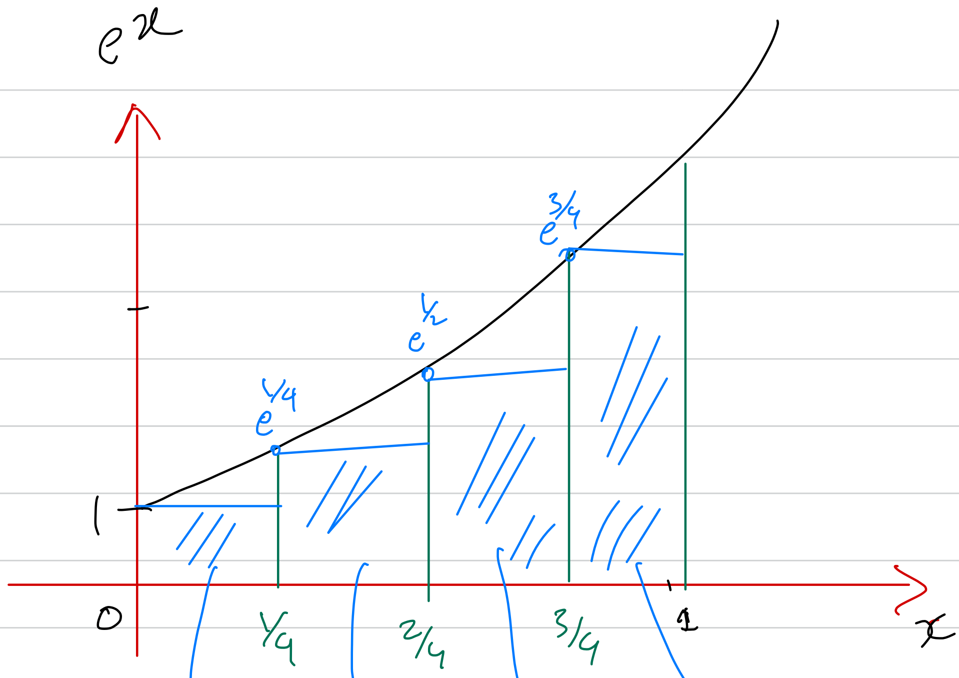
(Derivative of area) $\rightarrow C'(x) = 6-x \leftarrow$ (function)

2

(a)

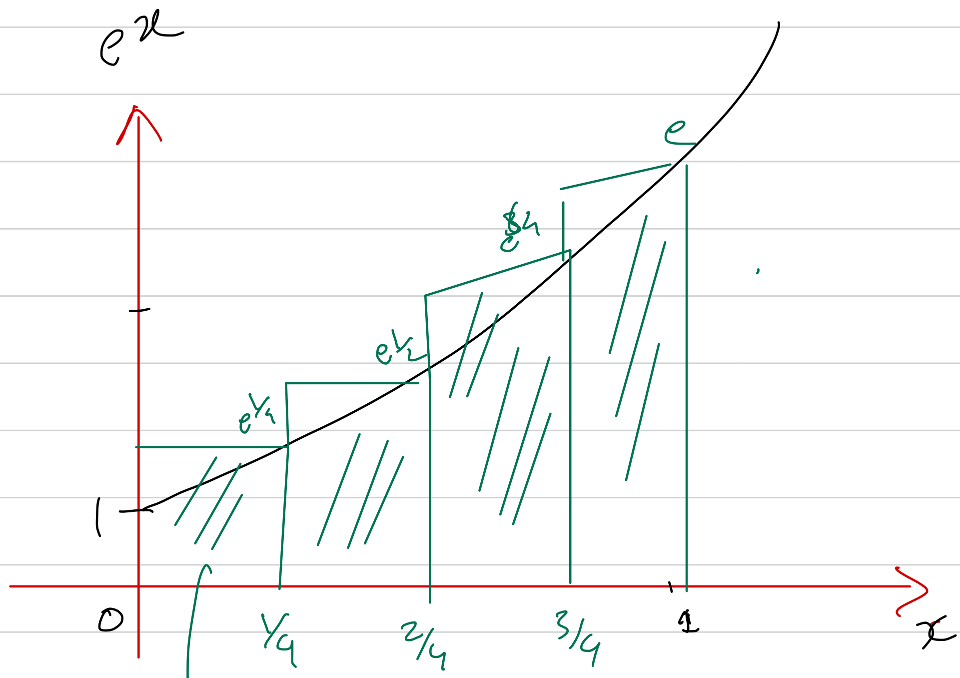


(b)



$$L_1 = \frac{1}{4}\cdot 1 + \frac{1}{4}\cdot e^{1/4} + \frac{1}{4}\cdot e^{1/2} + e^{3/4}\cdot\frac{1}{4} = 1.51$$

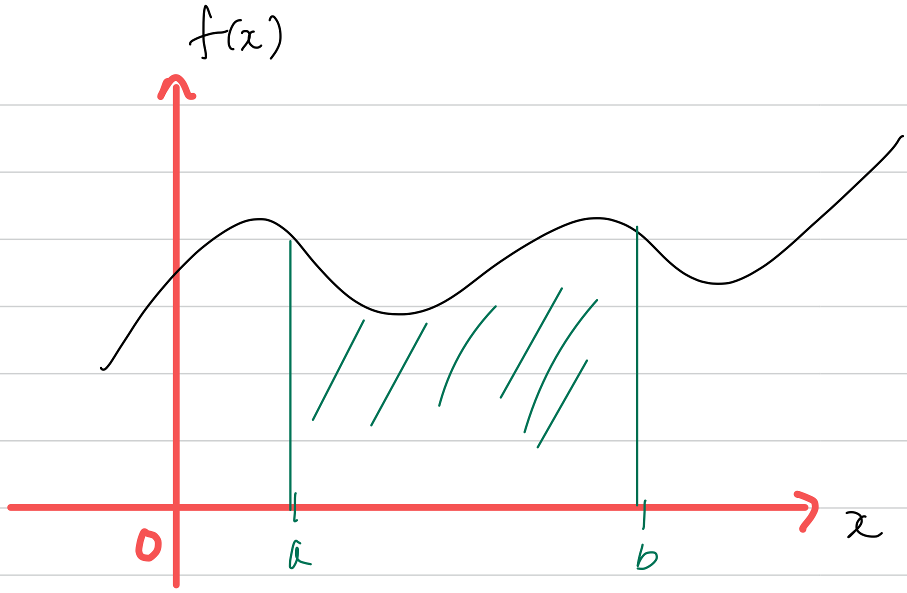
(c)



$$U_1 \quad \frac{1}{4}\cdot e^{1/4} + \frac{1}{4} e^{1/2} + \frac{1}{4} e^{3/4} + \frac{1}{4}\cdot e \approx 1.942$$

(d)

$$\left.\begin{aligned} \frac{U_1 + L_1}{2} &= 1.72 \\ e - 1 &= 1.71 \end{aligned}\right\} \text{approximation}$$

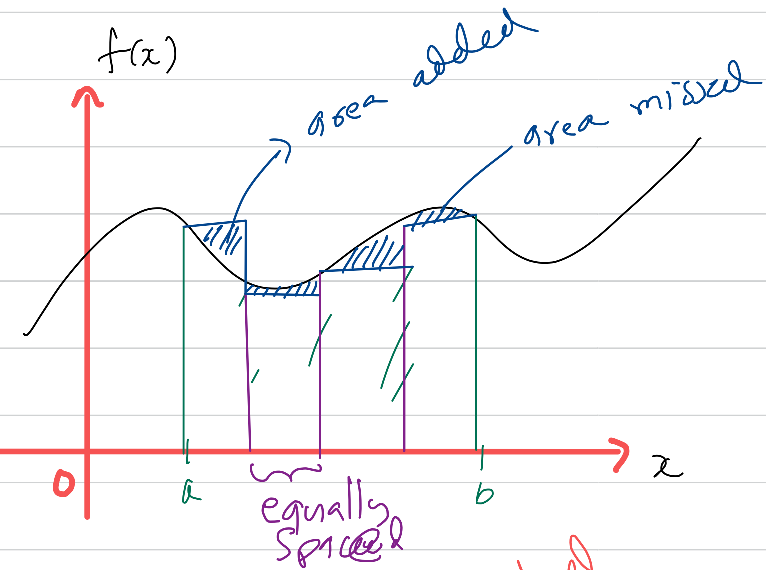


$f : [a,b] \to [0,\infty)$

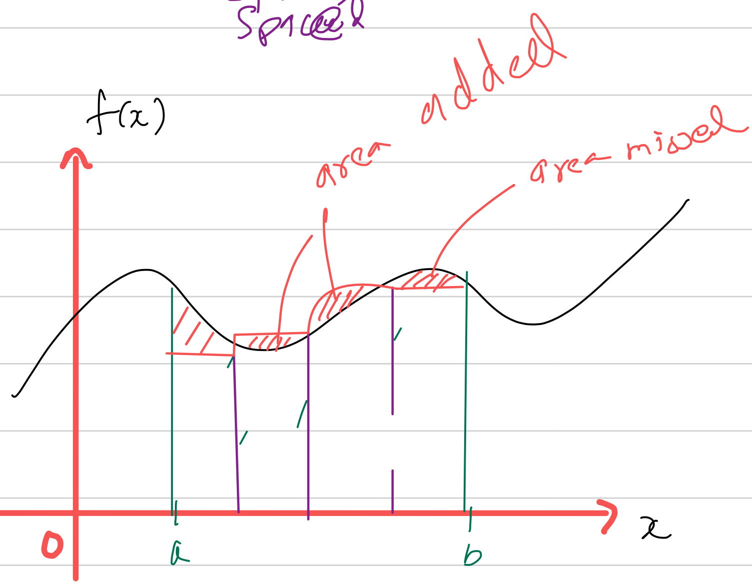
$\int_a^b f(x)\,dx \equiv$ Area under the curve f between a and b

Area Approximation

I



II



Approximation is better if there are more bins

Approach I

$$P_2(x) = \alpha x^2 + \beta x + \gamma$$

Find α, β, γ

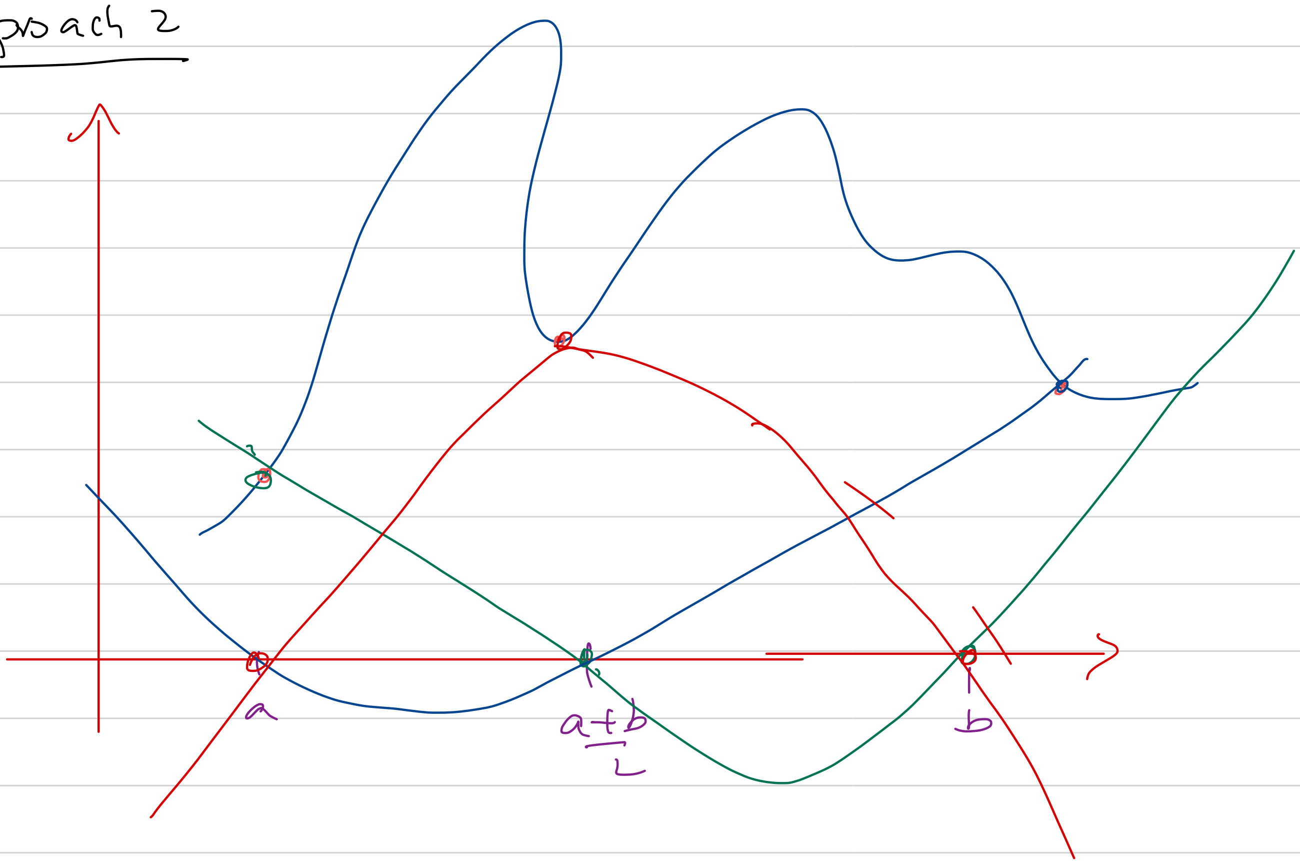
$\boxed{\text{Solve}}$ $\leftarrow$

$$f(a) = \alpha a^2 + \beta a + \gamma$$

$$f(b) = \alpha b^2 + \beta b + \gamma$$

$$f\left(\frac{a+b}{2}\right) = \alpha\left(\frac{a+b}{2}\right)^2 + \beta\left(\frac{a+b}{2}\right) + \gamma$$

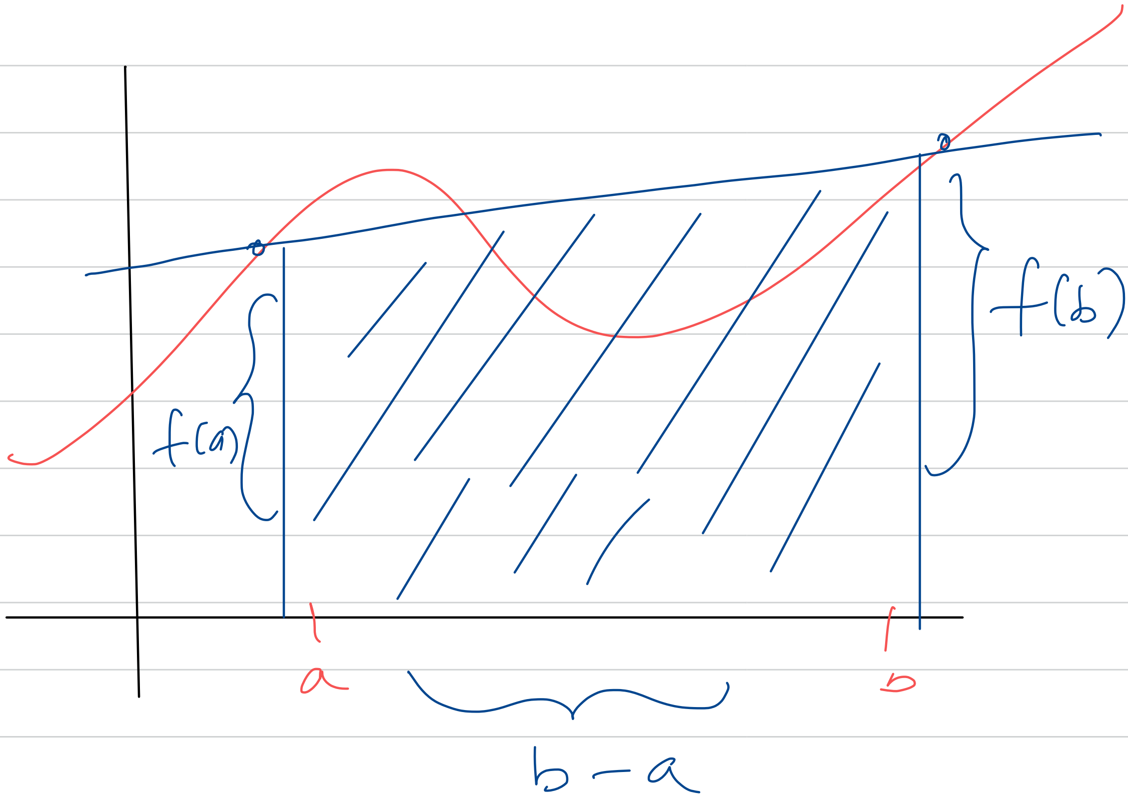
Approach 2



$$P_2(x) = \frac{(x-b)\left(x-\frac{a+b}{2}\right)f(a)}{(a-b)\left(a-\frac{a+b}{2}\right)} + \frac{(x-a)\left(x-\frac{a+b}{2}\right)f(b)}{(b-a)\left(b-(a+b)/2\right)}$$

$$+ \frac{(x-a)(x-b)}{\left(\frac{a+b}{2}-a\right)(\quad -b)} f\left(\frac{a+b}{2}\right)$$

$$\int_a^b P_1(x)\,dx = ? \quad \text{area under } P_1(\cdot)$$



$$= (b-a)\frac{\left(f(a)+f(b)\right)}{2}$$