1. Let $a, b : \mathbb{N} \to \mathbb{R}_+$ be two sequences
   - $a_n = O(b_n)$ if there exists $N_0 \in \mathbb{N}$ and $c > 0$ such that $a_n \leq cb_n$ for all $n \geq N_0$
   - $a_n = o(b_n)$ if for every $\epsilon > 0$ there exists N_0 such that $a_n \leq \epsilon b_n$ for all $n \geq N_0$
2. Consider the following sets of sequences:
   (a) $a_n = n^3 + 5n^2 + 15$ and $b_n = n^3 + 7n + 8$
   (b) $a_n = n^3 + 5n^2 + 15$ and $b_n = n^3 + 700000n + 1000$
   (c) $a_n = n^3 + 5n^2 + 15$ and $b_n = 0.0005n^4 + 7n^2 + 8$
   (d) $a_n = n^3 + 5n^2 + 15$ and $b_n = 2^n$
      i. The below R-code is available in Dropbox shared folder BIGO.R.

```
> #writing sequences as functions
> a <- function(n){ n^3+ 5*n^2+ 15}
> b1 <- function(n){n^3+ 7*n+ 8}
> b2 <- function(n){n^3+ 700000*n+ 1000}
> b3 <- function(n){0.0005*n^4+ 7*n^2+ 8}
> b4 <- function(n){2^n }
> # setting the number of steps
> n = seq(1, 100, by = 1)
> #calculating the ratio of sequences
> c1  <-  a(n)/b1(n)
> c2  <-  a(n)/b2(n)
> c3  <-  a(n)/b3(n)
> c4  <-  a(n)/b4(n)
> #setting chart in plot to four spaces
> par(mfrow= c(2,2))
> #plotting the sequences in one chart
> plot(c1~n, cex= 0.2, col= "#d55e00", xlab= "n", ylab= "a/b1", main = "(a)")
> plot(c2~n, cex= 0.2, col= "#cc79a7", xlab= "n", ylab= "a/b2", main = "(b)")
> plot(c3~n, cex= 0.2, col= "#0072b2", xlab= "n", ylab= "a/b3", main = "(c)")
> plot(c4~n, cex= 0.2, col= "#009e73", xlab= "n", ylab= "a/b4", main = "(d)")
>
```

      In R-studio cloud or elsewhere please run the above code to obtain the plots.
      ii. From the plots can you guess for each of (a), (b), (c), (d), if $a_n = O(b_n)$ or $a_n = o(b_n)$.
      iii. Change the R and plot till $n = 5000$ for the sequences mentioned in (a), (b), (c), and (d).
      iv. Decide (with proof) whether $a_n = O(b_n)$ or $a_n = o(b_n)$ for sequences in (a), (b), (c), (d).
3. For each of the following indicate whether $a_n = O(b_n)$, or $a_n = o(b_n)$
   (a) $a_n = n^3 + 5n^2 + 15$ and $b_n = n^3 + 7n + 8$
   (b) $a_n = nb^n$, for $b \in (0, 1)$ and $b_n = \frac{1}{n^4}$