



June 25<sup>th</sup>, 2021



WRAP UP

in

R

The screenshot displays the RStudio Cloud interface. The top browser tabs include 'Probability authors/titles', '(4) WhatsApp', 'Siva Athreya's Home Page', and 'RStudio Cloud'. The browser address bar shows 'rstudio.cloud/project/2596313'. The RStudio interface includes a menu bar (File, Edit, Code, View, Plots, Session, Build, Debug, Profile, Tools, Help), a toolbar, and a sidebar with 'Your Workspace' and 'Learn' sections.

The main editor window shows the following R code:

```
1 x <- seq(-20, 20, length = 100)
2 y <- seq(-20, 20, length = 100)
3 f = function(x,y){x^2+y^2}
4 z = outer(x,y,f)
5 require(plot3D)
6 par(mfrow=c(1,2))
7 persp3D(x,y,z, theta = 30)
8 persp3D(x,y,z, theta = 90)
9
```

The Environment pane shows the following data:

Object	Value
Data	
z	num [1:100, 1:100] 800 784 768 753 738 ...
Values	
x	num [1:100] -20 -19.6 -19.2 -18.8 -18.4 ...
y	num [1:100] -20 -19.6 -19.2 -18.8 -18.4 ...
Functions	
f	function (x, y)

The Plots pane shows two 3D surface plots of the function  $z = x^2 + y^2$ . The left plot is a perspective view with  $\theta = 30^\circ$ , and the right plot is a perspective view with  $\theta = 90^\circ$ . Both plots show a paraboloid shape with a color scale from 0 (blue) to 800 (red).

The Console pane shows the following output:

```
Content type 'application/x-tar' length 2793293 bytes (2.7 MB)
downloaded 2.7 MB

* installing *binary* package 'misc3d' ...
* DONE (misc3d)
* installing *binary* package 'plot3D' ...
* DONE (plot3D)

The downloaded source packages are in
'/tmp/Rtmp1sEdst/downloaded_packages'
> require(plot3D)
Loading required package: plot3D
Warning message:
In fun(libname, pkgname) : couldn't connect to display ':0'
> par(mfrow=c(1,2))
> persp3D(x,y,z, theta = 30)
> persp3D(x,y,z, theta = 90)
```

Plan:-

6:05 - 7:30 { ... Review 2 problems  
 10 volunteers ← ... WRAP UP in R

7:30 - 7:45 - 3 videos

7:45 - 8 - Feedback form

8 - { Purvi, Anita, Siva | Next steps

(d)  $a_n = n^3 + 5n^2 + 15$        $b_n = 2^n$       

Guess:-  $a_n = o(b_n)$

$$a_n = n^3 + 5n^2 + 15 \geq 1 + 5 + 15 = 21 \quad \forall n \geq 1$$

$$b_n = 2^n \geq 2 \quad \forall n \geq 1$$

Lower bound -

$$0 \leq \frac{a_n}{b_n}$$

let  $\epsilon > 0$  be given

Objective:- To find  $N \geq 1$  st.

$$\frac{a_n}{b_n} < \epsilon \quad \forall n \geq N$$

$$\frac{a_n}{b_n} = \frac{n^3 + 5n^2 + 15}{2^n}$$

- Idea:-
- expand -
  - $2^n$  growing fast
  - $2^n \geq n^3 + 5n^2 + 15 \dots$  (May not be enough)
  - [Induction!]

$n \geq 1$  [Inductively]

Show :-  $2^n = (1+1)^n = \sum_{k=0}^n \binom{n}{k} \geq c n^4$  for some  $c > 0$   
 $\forall n \geq 4$

$$\frac{a_n}{b_n} \leq \frac{n^3 + 5n^2 + 15}{c n^4} = \frac{1}{c} \left[ \frac{1}{n} + \frac{5}{n^2} + \frac{15}{n^3} \right]$$

$\forall n \geq 4$

Yesterday's      Subst do 2

$\exists N \geq 1$  st

$$\frac{1}{c} \left[ \frac{1}{n} + \frac{5}{n^2} + \frac{15}{n^3} \right] < \epsilon$$

... finish the proof like yesterday  
 ...  $a_n = o(b_n)$

Proof of show :-

$$\begin{aligned} 2^n &= \sum_{k=0}^n \binom{n}{k} \\ &= \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \binom{n}{4} + \dots + \binom{n}{n} \\ &\geq 0 + \binom{n}{4} + 0 \\ &= \frac{n(n-1)(n-2)(n-3)}{24} \end{aligned}$$

$$= n^4 \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \left(1 - \frac{3}{n}\right) \frac{1}{24}$$

; | | | |

$n \geq 4$

$$\begin{array}{cccccc} \downarrow & \vdots & \vdots & \vdots & \downarrow & \\ n^4 & \sqrt[3]{1} & \sqrt[2]{1} & \sqrt[4]{1} & \sqrt[24]{1} & \end{array}$$

$= C n^4$  where  $C = \frac{1}{24} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{3}{4}$